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INVESTIGATION OF ISOTHERMAL, COMPRESSIBLE FLOW ACROSS A ROTATING SEALING DAM

I — ANALYSIS

by John Zuk and Lawrence P. Ludwig

Lewis Research Center
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ABSTRACT

The flow across a parallel sealing dam of the type that appears in gas turbine seals is analyzed for steady, laminar, subsonic, isothermal compressible flow with relative rotation of the sealing dam surfaces. Rotational effects on velocity profiles, pressure profiles, and other physical quantities of interest are found. Conditions are given under which a hydrostatic analysis of the radial pressure flow is a valid approximation. The error in estimating the mass leakage by using the hydrostatic radial flow formula is given.

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SUMMARY

The flow across a parallel sealing dam of the type that appears in gas turbine seals is analyzed for steady, laminar, subsonic, isothermal compressible flow with relative rotation of the sealing dam surfaces. Rotational effects on velocity profiles, pressure profiles, and other physical quantities of interest are found. The centrifugal force effect on the radial pressure flow is shown to be significant only if the radial pressure differential is very small or the speed is high. A hydrostatic analysis of the radial pressure flow is shown to be a valid approximation under the following conditions: (1) The sealing dam mean radius is much greater than the sealing dam radial width which, in turn, is much greater than the film thickness. (2) The radial pressure differential is sufficiently large. The error in estimating the mass leakage by using the hydrostatic radial-flow formula is given.

The equations presented are valid for the laminar, isothermal flow regime. The rotational flow analysis is valid until transition to turbulent flow. The radial pressure flow analysis is valid until the Mach number approaches $1/\sqrt{\gamma}$ (γ = specific-heat ratio), which is the limit of isothermal compressible flow.

A computer program to automate this analysis is presented in a companion report, II - COMPUTER PROGRAM.

INTRODUCTION

Some powerplants, such as advanced jet engines, exceed the operating limits of face contact seals (refs. 1 and 2). As a result, noncontact face seals are becoming necessary. A noncontact face seal which is pressure (force) balanced is shown in figure 1. In this seal the pressure drop occurs across a narrowly spaced sealing dam, and the axial force associated with this pressure drop is balanced by a predetermined hydrostatic

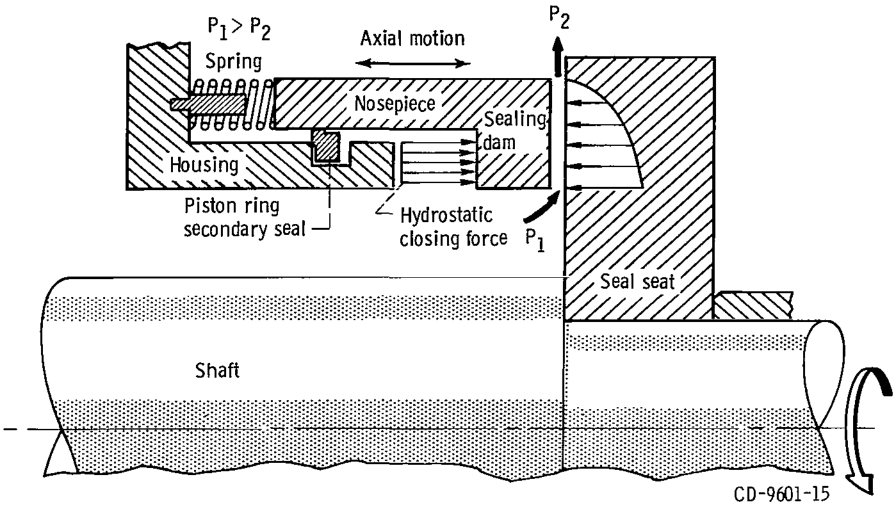


Figure 1. - Schematic of pressure balanced face seal, no axial film stiffness for parallel sealing surfaces.

closing force and a spring force. However, this configuration, with the sealing dam formed by parallel surfaces has an inherent problem, which is that the force due to the pressure drop across the sealing dam is independent of film thickness; hence, there is no way of maintaining a preselected film thickness which will allow tolerable leakage and still have noncontact operation. Since the force is independent of film thickness, the design also lacks axial film stiffness for sufficient dynamic tracking of the stationary nosepiece with the rotating seal seat. The seal nosepiece must follow the seal seat surface under different operating conditions without surface contact or excessive increase in film thickness, which would yield high leakage. Some of these operating conditions are axial runout, misalinement, and thermal deformation (coning and dishing).

A promising method of maintaining a preselected film thickness and achieving axial film stiffness is to add a gas bearing, such as a shrouded Rayleigh step pad bearing, to the conventional face seal (refs. 1 and 2). This is illustrated in figure 2. The axial sealing dam force associated with the pressure drop across the sealing dam, and the gas bearing force, are balanced by the hydrostatic and spring closing forces. The gas bearing has a desirable characteristic whereby the force increases with decreasing film thickness. If the seal is perturbed in such a way as to decrease the gap, the additional force generated by the gas bearing will open the gap to the original equilibrium position. In a similar manner, if the gap becomes larger, the gas bearing force decreases, and the closing force will cause the seal gap to return to the equilibrium position. Since a proper balance of the opening and closing forces must be found in order to determine a gap with a tolerable mass leakage, physical quantities of interest, such as pressure distribution and mass leakage, must be evaluated.

The pressure distribution and mass leakage have been calculated for the parallel

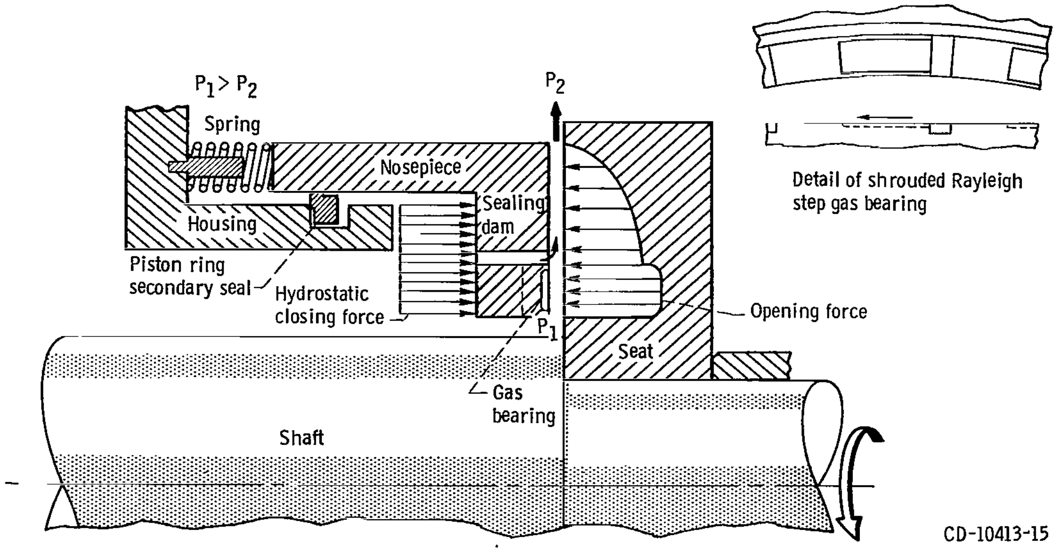


Figure 2. - Schematic of a pressure balanced face seal with a gas bearing added to obtain axial film stiffness.

film hydrostatic case; mathematical solutions for the hydrostatic, isothermal, compressible, viscous flow sealing dam exist in the literature (e.g., see Gross (ref. 3)). Carothers (ref. 4) has conducted compressible flow experiments on the radial flow of air between two closely spaced parallel plates; the pressure distribution was found for both subsonic and supersonic axial entrance flows. Grinnell (ref. 5) has theoretically and experimentally investigated compressible flow in a thin passage and has shown excellent agreement between theory and experiment. The effects of rotation of the seal seat with respect to the seal nosepiece (sealing dam surfaces) on the radial pressure flow, however, have been neglected.

This study was conducted (1) to investigate theoretically the effect of sealing dam rotation on mass leakage, pressure and velocity distributions, net pressure force, and center of pressure (radial direction) for an isothermal, compressible, viscous flow with parallel sealing dam surfaces; (2) to determine under what conditions, if any, the hydrostatic radial flow analysis is sufficient for engineering accuracy.

ANALYSIS

Basic Model

The sealing dam model, as shown in figures 3 and 4(a), consists of two parallel, concentric, circular rings in relative rotation at a constant speed separated by a very

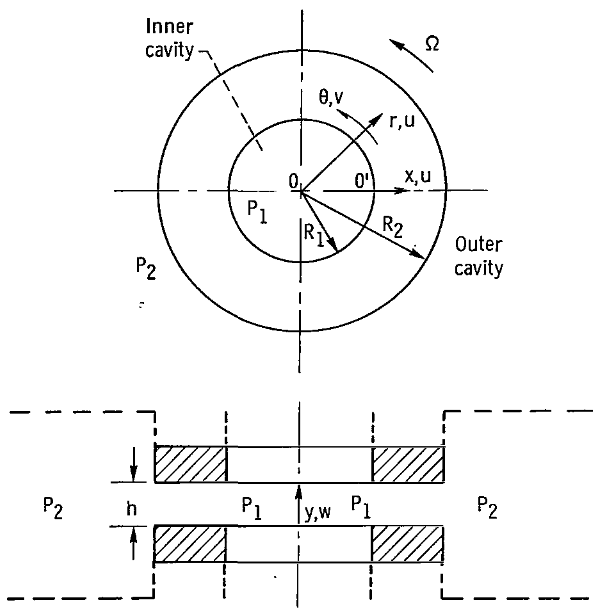


Figure 3. - Model of sealing dam.

narrow gap. A pressure differential exists between the rings' inner and outer radii. The fluid velocities are small in both the inner-diameter cavity and outer-diameter cavity which bound the sealing dam.

The model formulation is based on the following physical conditions:

(1) The fluid is homogeneous, compressible, viscous, and Newtonian.

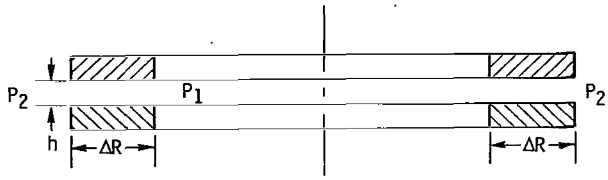
(2) The flow is steady and laminar (continuum flow regime), and the body forces are negligible.

(3) The bulk modulus is ignored ($\lambda = -2/3\ \mu$). (All symbols are defined in appendix B.) This is Stokes idealization (ref. 3). This condition will be valid unless the gas is under high pressure, very dense (e.g., shock-wave structure), or rarefied.

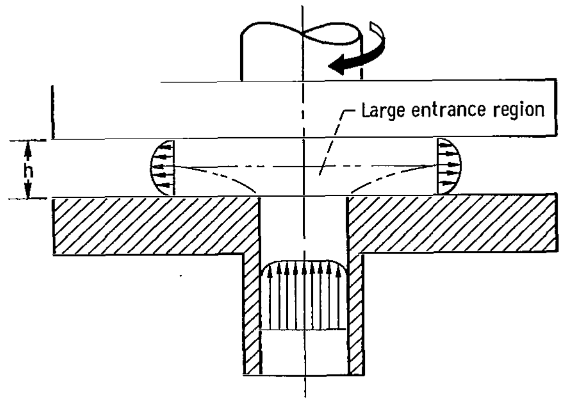
(4) The fluid behaves as a perfect gas.

(5) Since ΔR is much greater than h, the entrance region effects are neglected; hence, the convective inertia forces are neglected. This means that the seal is treated as operating entirely in the viscous region. This case is contrasted to the case where the gap size is large, as illustrated in figure 4(b) for a radial diffuser.

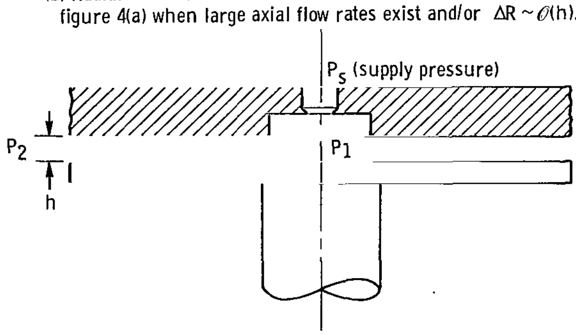
(6) The fluid film is isothermal. This means that all heat generated in the film is conducted away through the walls. This is a standard assumption of lubrication theory. The validity of this assumption breaks down for cases of large thermal gradients in the sealing dam and when the frictional heating is high (e.g., small gap or high speed). However, thermal analysis of a seal (unpublished work by T. E. Russell of Lewis) shows that the sealing dam can be closely approximated by a constant temperature. In any case, the Mach number must be less than $1/\sqrt{\gamma}$. This is the limit of the validity of isothermal



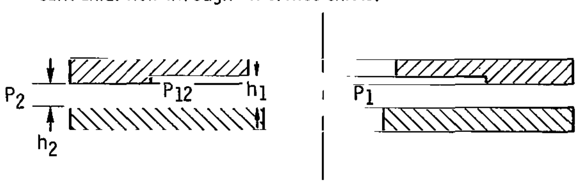
(a) Sealing dam model used in analysis. Model characteristics: no axial flow; narrow gap, $h \ll \Delta R$.



(b) Radial diffuser model. Model does not conform to model of figure 4(a) when large axial flow rates exist and/or $\Delta R \sim \mathcal{O}(h)$.



(c) Hydrostatic or externally pressurized thrust bearing model. Model does not conform to model of figure 4(a) when significant axial flow through an orifice exists.



(d) Radial step seal model. Model of figure 4(a) is valid when $h_2 \gg h_1$ (then $P_{12} \cong P_1$) and inner cavity velocity is not significant.

Figure 4. - Various seal and bearing models illustrating conditions of analysis validity.

duct flow analyses as stated in most gas dynamics textbooks (e.g., Shapiro (ref. 6)).

(7) The entrance Mach number is close to zero. This means that there cannot be a large axial flow source on one surface impinging on the radial surface, as in a hydrostatic bearing (fig. 4(c)).

(8) The fluid velocity in the reservoir is considered to be negligible (stagnant), and thus its effects are neglected in this analysis. This model is, therefore, not valid for the radial step seal shown in figure 4(d) when radial velocities are significant.

GOVERNING EQUATIONS

The governing flow equations for a compressible fluid with constant viscosity in vector notation are (ref. 7)

Conservation of mass:

$$\frac{D\rho}{Dt} + \rho\vec{\nabla} \cdot \vec{V} = 0$$

Conservation of momentum (Navier-Stokes equations):

$$\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla}P - \mu\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) + (\lambda + 2\mu)\, \vec{\nabla}\, (\vec{\nabla} \cdot \vec{V}) + \vec{F}$$

Equation of state for an isothermal process:

$$P = P(\rho)$$

By using cylindrical coordinates (fig. 3) and by applying the conditions assumed in the mathematical model, with the exception of the narrow gap approximation, the governing flow equations reduce to

Conservation of mass:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r u) + \frac{\partial}{\partial y}(\rho w) = 0 \qquad (1)$$

Conservation of momentum:

(1) Radial direction

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial y} - \frac{v^2}{r}\right) = -\frac{\partial P}{\partial r} + \mu\left[\frac{4}{3}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3}\frac{\partial^2 w}{\partial r\,\partial y}\right] \tag{2}$$

(2) Azimuthal direction

$$\rho\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial y} + \frac{uv}{r}\right) = \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial y^2} - \frac{v}{r^2}\right) \tag{3}$$

(3) Axial direction

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{4}{3}\frac{\partial^2 w}{\partial y^2} + \frac{1}{3}\left(\frac{1}{r}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y\,\partial r}\right)\right] \tag{4}$$

Equation of state:

$$\frac{P}{\rho} = \mathcal{R}T = \text{Constant} \tag{5}$$

Note that in equations (1) to (5) the θ-dependent terms have dropped out because of rotational symmetry.

Nondimensionalize the preceding set of equations in the following manner:

$$r^* = \frac{r}{\Delta R} \qquad P^* = \frac{P}{\rho_0 U^2}$$

$$y^* = \frac{y}{h} \qquad \rho^* = \frac{\rho}{\rho_0}$$

$$u^* = \frac{u}{U} \qquad Re_h = \frac{\rho_0 U h}{\mu}$$

$$v^* = \frac{v}{V}$$

$$w^* = \frac{w}{W}$$

The continuity equation becomes

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}(\rho^* r^* u^*) + \frac{W}{U}\frac{\Delta R}{h}\frac{\partial}{\partial y^*}(\rho^* w^*) = 0 \tag{6}$$

To have all the terms of the equation the same order of magnitude for this first-order approximation, $(W/U)(\Delta R/h)$ must be of the order of 1. Hence, $W = U(h/\Delta R)$ and, therefore, $w = w^* U(h/\Delta R)$. In this analysis, since $\Delta R \gg h$ (narrow gap approximation and away from entrance region), w will be smaller than u and will be negligible to the order of this analysis. Since $w(o) = 0$ and $w(h) = 0$, $w \equiv 0$. Hence, from the axial momentum equation (eq. (4)), $\partial P/\partial y \cong 0$; and the continuity equation becomes

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}(\rho^* r^* u^*) = 0 \tag{6a}$$

The dimensionless radial momentum equation becomes

$$\rho^* u^* \frac{\partial u^*}{\partial r^*} - \left(\frac{V}{U}\right)^2 \rho^* \frac{v^{*2}}{r^*} = -\frac{\partial P^*}{\partial r^*}$$

$$+ \frac{1}{Re_h}\left(\frac{4}{3}\frac{h}{\Delta R}\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{4}{3}\frac{h}{\Delta R}\frac{1}{r^*}\frac{\partial u^*}{\partial r^*} - \frac{4}{3}\frac{h}{\Delta R}\frac{u^*}{r^{*2}} + \frac{\Delta R}{h}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{1}{3}\frac{h}{\Delta R}\frac{\partial^* w^*}{\partial r^*\,\partial y^*}\right) \tag{7}$$

Again use the narrow gap approximation ($h \ll \Delta R$) and neglect the entrance region. Thus, convective inertia forces are neglected. This results in $Re_h (h/\Delta R) \ll 1$.

The dimensionless radial momentum equation becomes

$$-\left(\frac{V}{U}\right)^2 \rho^* \frac{v^*}{r^*} = -\frac{\partial P^*}{\partial r^*} + \frac{1}{Re_h}\left(\frac{\Delta R}{h}\right)\frac{\partial^2 u^*}{\partial y^{*2}} \tag{8}$$

Note if $V/U < 1/\sqrt{Re_h (h/\Delta R)}$, the radial pressure flow can be treated as uncoupled from the rotational shear flow.

The dimensionless circumferential momentum equation becomes

$$\rho^*\left(u^* \frac{\partial v^*}{\partial r^*} + \frac{u^* v^*}{r^*}\right) = \frac{1}{Re_h}\left[\left(\frac{h}{\Delta R}\right)\frac{\partial^2 v^*}{\partial r^{*2}} + \left(\frac{h}{\Delta R}\right)\frac{1}{r^*}\frac{\partial v^*}{\partial r^*} + \left(\frac{\Delta R}{h}\right)^2 \frac{\partial^2 v^*}{\partial y^{*2}} - \left(\frac{h}{\Delta R}\right)\frac{v^*}{r^{*2}}\right] \tag{9}$$

Again for $\Delta R >> h$ and $Re_h (h/\Delta R) << 1$ equation (9) becomes

$$\frac{\partial^2 v^*}{\partial y^{*2}} = 0 \tag{10}$$

Now in dimensional form the equations to be solved are
Continuity (from eq. (6a)):

$$\frac{\partial}{\partial r} (\rho r u) = 0 \tag{11}$$

This form of the continuity equation is not used but is replaced by the integrated form, which is shown to be the conservation of mass flow in the radial direction.
Radial momentum (from eq. (8)):

$$-\frac{\rho v^2}{r} = -\frac{\partial P}{\partial r} + \mu \frac{\partial^2 u}{\partial y^2} \tag{12}$$

The centripetal inertia term on the left side is ignored in the compressible Reynolds' lubrication equation.
Circumferential momentum (from eq. (10)):

$$\frac{\partial^2 v}{\partial y^2} = 0 \tag{13}$$

Solving equation (13) yields

$$v = C_1(r)y + C_2(r)$$

Applying the boundary conditions,

$$v = 0 \rightarrow C_2 = 0 \qquad \text{at } y = 0$$

$$v = r\Omega \rightarrow C_1 = \frac{r\Omega}{h} \qquad \text{at } y = h$$

Hence,

$$v = \frac{r\Omega y}{h} \tag{14}$$

Now solve equation (12) by substituting for the circumferential velocity equation (eq. (14)) and the perfect gas law (eq. (5)).

$$\rho = \frac{\rho_0 P}{P_0} = \frac{P}{\mathcal{R}T}$$

Since the circumferential and axial variations are shown to be negligible, $P = P(r)$, and equation (12) becomes

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dP}{dr} - \frac{\rho_0 \Omega^2}{P_0 h^2 \mu} r y^2 P$$

By integrating twice with respect to y

$$u = \frac{1}{2\mu}\frac{dP}{dr} y^2 - \frac{\rho_0 \Omega^2}{12\,\mu P_0 h^2} r y^4 P + C_1' y + C_2'$$

Applying the boundary conditions

$$u = 0 \rightarrow C_2' = 0 \qquad \text{at } y = 0$$

$$u = 0 \rightarrow C_1' = -\frac{h}{2\mu}\frac{dP}{dr} + \frac{\rho_0 \Omega^2 h r P}{12\mu P_0} \qquad \text{at } y = h$$

Thus,

$$u = \frac{1}{2\mu}\frac{dP}{dr}(y^2 - hy) - \frac{\rho_0 \Omega^2 r P}{12\mu P_0 h^2}(y^4 - h^3 y) \tag{15}$$

The mass flow at any radius is

$$\dot{M} = 2\pi r\rho \int_0^h u\,dy = -\frac{\pi h^3 \rho r}{6\mu}\frac{dP}{dr} + \frac{\pi\rho_0\Omega^2 h^3 \rho r^2 P}{20\,\mu P_0} \tag{16}$$

Substituting the perfect gas law (eq. (5)) into equation (16) yields

$$\dot{M} = -\left(\frac{\pi h^3 \rho_0}{12\,\mu P_0}\right) r \frac{dP^2}{dr} + \left(\frac{\pi \rho_0^2 \Omega^2 h^3}{20\,\mu P_0^2}\right) r^2 P^2 \tag{17}$$

or

$$\frac{dP^2}{dr} - \left(\frac{3}{5}\,\frac{\rho_0\Omega^2}{P_0}\right) r P^2 = -\left(\frac{12\,\mu P_0 \dot{M}}{\pi\rho_0 h^3}\right)\frac{1}{r} \tag{18}$$

To facilitate the solution, nondimensionalize equation (18) in the following manner: Let

$$\tilde{P}^2 = \frac{P^2}{P_2^2}$$

$$\dot{M}^* = \frac{-12\,\mu\dot{M}\mathcal{R}T}{\pi h^3 P_2^2}$$

$$\tilde{r} = \frac{r - R_2}{R_2}$$

This results in the equation

$$\frac{d\tilde{P}^2}{d\tilde{r}} - \left(\frac{3\rho_0 R_2^2 \Omega^2}{5P_0}\right)(1 + \tilde{r})\ P^2 = \frac{\dot{M}^*}{1 + \tilde{r}} \tag{19}$$

Now let

$$\tilde{X} = 1 + \tilde{r} = \frac{r}{R_2}$$

$$K_1 = -\frac{3R_2^2\Omega^2}{5\mathcal{R}T}$$

Thus, equation (19) becomes

$$\tilde{X}\frac{d\tilde{P}^2}{d\tilde{X}} + K_1\tilde{X}^2\tilde{P}^2 = \dot{M}^* \tag{20}$$

The conservation of mass must be satisfied

$$\frac{d\dot{M}^*}{d\tilde{X}} = 0 \tag{21}$$

(which replaces the continuity eq. (11)) or

$$\frac{d}{d\tilde{X}}\left(\tilde{X}\frac{d\tilde{P}^2}{d\tilde{X}}\right) + K_1\frac{d}{d\tilde{X}}\left(\tilde{X}^2\tilde{P}^2\right) = 0 \tag{22}$$

Equation (22) can be solved subject to the following boundary conditions:

$$\left.\begin{aligned} \tilde{X} &= \tilde{X}_1 = \frac{R_1}{R_2} \\ \tilde{P}^2 &= \tilde{P}_1^2 \end{aligned}\right\} \text{at } r = R_1$$

$$\left.\begin{aligned} \tilde{X} &= 1 \\ \tilde{P}^2 &= \tilde{P}_2^2 = 1 \end{aligned}\right\} \text{at } r = R_2$$

Equation (19) is a first-order nonhomogeneous ordinary differential equation with variable coefficients and can be solved by using the integrating factor method

$$\tilde{P}^2 \cdot \exp\left[\int A(\tilde{X})d\tilde{X}\right] = \int B(\tilde{X}) \cdot \exp\left[\int A(\tilde{X})d\tilde{X}\right] d\tilde{X} + C$$

Here $A(\tilde{X}) = K_1\tilde{X}$ and $B(\tilde{X}) = \dot{M}^*/\tilde{X}$
Thus,

$$\tilde{P}^2 \cdot \exp\left(\frac{K_1\tilde{X}^2}{2}\right) = \dot{M}^* \int \frac{\exp\left(\frac{K_1\tilde{X}^2}{2}\right)}{\tilde{X}} d\tilde{X} + C$$

Now apply the boundary condition

$$\left.\begin{array}{l} \tilde{X} = 1 \\ \tilde{P}^2 = \tilde{P}_2^2 = 1 \end{array}\right\} \text{ at } r = R_2$$

Thus,

$$C = \tilde{P}_2^2 \exp\left(\frac{K_1}{2}\right) - \dot{M}^* \int_{\tilde{X}}^{1} \exp\left(\frac{K_1\tilde{X}^2}{2}\right) d\tilde{X}$$

and

$$\tilde{P}^2 = \dot{M}^* \exp\left(\frac{-K_1\tilde{X}_2}{2}\right) \int_{\tilde{X}}^{1} \frac{\exp\left(\frac{-K_1\tilde{X}_2}{2}\right)}{\tilde{X}} d\tilde{X} + \exp\left[\frac{-K_1}{2}(\tilde{X}^2 - 1)\right] \tilde{P}_2^2 \qquad (23)$$

Solve for the mass flow by applying the second boundary condition to equation (23)

$$\left.\begin{aligned} \tilde{X} &= \tilde{X}_1 \\ \tilde{P}^2 &= \tilde{P}_1^2 \end{aligned}\right\} \text{ at } r = R_1$$

This boundary condition yields the following mass flow equation in dimensional form:

$$\dot{M} = - \frac{\pi h^3 \rho_2 \left\{ \exp\left[\frac{-K_1}{2} \left(\frac{R_1^2}{R_2^2} - 1 \right) \right] P_2^2 - P_1^2 \right\} \exp\left(\frac{K_1}{2} \frac{R_1^2}{R_2^2} \right)}{12\, \mu P_2 \displaystyle\int_{R_1/R_2}^{1} \frac{\exp\left(\frac{K_1}{2} \tilde{X}^2 \right)}{\tilde{X}}\, d\tilde{X}} \tag{24}$$

Substitute equation (24) into equation (23), which results in the square pressure distribution equation

$$P_2 = - \frac{\displaystyle\int_{r/R_2}^{1} \frac{\exp\left(\frac{K_1 \tilde{X}}{2} \right)}{\tilde{X}}\, d\tilde{X}}{\displaystyle\int_{R_1/R_2}^{1} \frac{\exp\left(\frac{K_1 \tilde{X}^2}{2} \right)}{\tilde{X}}\, d\tilde{X}} \exp\left[\frac{-K_1 \left(\frac{r^2}{R_2^2} - \frac{R_1^2}{R_2^2} \right)}{2} \right] \left\{ \exp\left[\frac{-K_1}{2} \left(\frac{R_1^2}{R_2^2} - 1 \right) \right] P_2^2 - P_1^2 \right\}$$

$$+ \exp\left[- \frac{K_1}{2} \left(\frac{r^2}{R_2^2} - 1 \right) \right] P_2^2 \tag{25}$$

The total force per unit width is found from

$$\frac{F}{L} = \int_0^{R_2 - R_1} (P - P_{min})\, dX \tag{26}$$

The center of pressure is

$$X_c = \frac{\int_0^{R_2 - R_1} (P - P_{min})\, X dX}{\frac{F}{L}} \tag{27}$$

where P_{min} is the smaller pressure of the two pressure boundary conditions.

In the computer program (see II - COMPUTER PROGRAM (ref. 8)), the integrations in equations (24) to (27) are performed numerically.

Since equation (26) does not depend on film thickness, there is no axial film stiffness for the parallel sealing dam surface case, and a gas bearing must be used to obtain axial film stiffness (see fig. 2).

Additional Parameters Calculated by Computer Program (Ref. 8)

The average radial velocity at any radial point x (see fig. 3) in the sealing dam gap is found from

$$u_{av}(x) = \frac{\dot{M}}{\rho(x) hL} = \frac{\dot{M} P_1}{L \rho_1 P(x) h} \tag{28}$$

The local Mach number at any x is then

$$M(X) = \frac{u_{av}(x)}{a} = \frac{u_{av}(x)}{\sqrt{\gamma \mathcal{R} T}} \tag{29}$$

where a is the speed of sound.

The pressure flow Reynolds number is determined by using the hydraulic radius 2h as the characteristic length

$$Re_h = \frac{P_{min} u_{av} 2h}{\mu \mathcal{R} T} \tag{30}$$

The Knudsen number can be found from (ref. 9)

$$Kn = \frac{\text{Molecular mean free path}}{\text{Mean film thickness}} \cong \frac{1.48\ M_{max}}{\frac{Re_{2h}}{2}} \tag{31}$$

Under conditions of very small film thicknesses, the Knudsen number may be greater than 0.01, and this continuum analysis would no longer be valid. (In that case, a slip flow regime analysis must be used.)

The total power is found by considering the viscous shear due to rotation only

$$\left.\begin{aligned} \text{Power} &= \overline{R}\Omega.\quad (\text{Shear force}) = \frac{\overline{R}^2 \Omega^2 \mu}{h} \int_A dA \\ \text{Power} &= \frac{\mu \overline{R}^2 \Omega^2 A}{h} \end{aligned}\right\} \tag{32}$$

where $\overline{R}\Omega$ is the mean rotational velocity.

A rough estimate of the gas temperature rise through the film can be found by assuming that all of the energy dissipated by viscous shear is added to the gas film

$$T_{film,\ av} - T = \frac{\mu \overline{R}^2 \Omega^2 A}{h C_p \dot{M}} \tag{33}$$

This calculated film temperature rise will be higher than actually occurs, since the predominant mode of heat transfer, conduction by the walls, is neglected.

The previously derived equations, placed in the form used in the computer program (see ref. 8), are shown in table I in the English system of units.

TABLE I. - FORM OF PERTINENT EQUATION FOUND IN COMPUTER PROGRAM IN REFERENCE 8

(IN SAME SEQUENCE AS EQUATIONS APPEAR IN PROGRAM)

$$A = \pi\left(R_2^2 - R_1^2\right),\ \text{in.}^2$$

$$\mathcal{R} = \frac{\underline{\mathcal{R}}}{M},\ \frac{\text{ft-lbf}}{(\text{lbm})(^{\circ}\text{R})}\quad \text{where } \underline{\mathcal{R}} = 1545.4\ \frac{\text{ft-lbf}}{(\text{lb-mole})(^{\circ}\text{R})}$$

$$\rho_1 = \frac{P_1\left(\dfrac{144\ \text{in.}^2}{\text{ft}^2}\right)}{\mathcal{R}(T + 460)\left[32.174\ \dfrac{(\text{lbm})(\text{ft})}{(\text{lbf})(\text{sec}^2)}\right]},\ \frac{(\text{lbf})(\text{sec}^2)}{\text{ft}^4}$$

$$a = \sqrt{\gamma\mathcal{R}(T + 460)\left[32.174\ \frac{(\text{lbm})(\text{ft})}{(\text{lbf})(\text{sec}^2)}\right]},\ \frac{\text{ft}}{\text{sec}}$$

$$\text{If } L = 0,\ L = 2\pi\,\frac{R_1 + R_2}{2},\ \text{in.}$$

$$\text{If } \rho_0 = 0,\ \rho_0 = \rho_1,\ \frac{(\text{lbf})(\text{sec}^2)}{\text{ft}^4}$$

$$\text{If } N \neq 0,\ V = \left(\frac{\pi N}{\dfrac{12\ \text{in.}}{\text{ft}}}\right)\left(\frac{R_1 + R_2}{2}\right)\left(\frac{\text{min}}{60\ \text{sec}}\right),\ \frac{\text{ft}}{\text{sec}}$$

If $N = 0$ and $V = 0$,

$\Delta T = 0,\ ^{\circ}\text{R}$

$C_p = 0,\ \dfrac{\text{Btu}}{(\text{lbm})(^{\circ}\text{R})}$

$$\text{Re}(\bar{R}) = \rho_0\left(\frac{R_1 + R_2}{2}\right)\left(\frac{\Omega}{\mu}\right)\left[\frac{h}{\left(\dfrac{60\ \text{sec}}{\text{min}}\right)\left(\dfrac{144\ \text{in.}^2}{\text{ft}^2}\right)}\right]$$

$$\Delta x = \frac{R_2 - R_1}{\text{Number of steps}},\ \text{in.}$$

$$\dot{M} = \frac{1.93\times10^3 h^3\rho_2\left\{P_1^2 - P_2^2\exp\left[-\dfrac{K_1}{2}\left(\dfrac{R_1^2}{R_2^2} - 1\right)\right]\right\}\exp\left(\dfrac{K_1}{2}\,\dfrac{R_1^2}{R_2^2}\right)}{\mu P_2\displaystyle\int_0^{R_2-R_1}\frac{\exp\left[\dfrac{K_1}{2}\left(\dfrac{x + R_1}{R_2}\right)^2\right]}{x + R_1}\,dx},\ \frac{\text{lbm}}{\text{min}}$$

$$\text{where } K_1 = -\frac{3R_2^2\Omega^2}{5\mathcal{R}T}$$

$$U_{av}(x) = \frac{\dfrac{144\ \text{in.}^2}{\text{ft}^2}}{\left[32.174\ \dfrac{(\text{lbm})(\text{ft})}{(\text{lbf})(\text{sec}^2)}\right]\left(\dfrac{60\ \text{sec}}{\text{min}}\right)}\,\frac{\dot{M}P_1}{L\rho_1 hP(x)},\ \frac{\text{ft}}{\text{sec}}$$

$$M = \frac{U_{av}}{a}$$

$$\text{Re}(P) = \frac{P_2U_{av}2h\left(\dfrac{12\ \text{in.}}{\text{ft}}\right)}{\left[32.174\ \dfrac{(\text{lbm})(\text{ft})}{(\text{lbf})(\text{sec}^2)}\right]\mu\mathcal{R}(T + 460)} = \text{Re}_{2h}$$

$$Q = 13.083\,\dot{M},\ \text{std cu ft/min}$$

$$\text{Power} = \left(\frac{\mu AV^2}{h}\right)\left(\frac{1\ \text{hp}}{\dfrac{550\ \text{ft-lbf}}{\text{sec}}}\right)\left(\frac{1}{\dfrac{12\ \text{in.}}{\text{ft}}}\right),\ \text{hp}$$

$$\Delta T = \frac{\left[42.42\ \dfrac{(\text{lbf})(\text{min})}{\text{hp}}\right](\text{Power})}{\dot{M}C_p},\ ^{\circ}\text{F}$$

$$H_{total} = 42.42(\text{Power}),\ \frac{\text{Btu}}{\text{min}}$$

$$F = L\int_0^{R_2-R_1}(P - P_{min})\,dx,\ \text{lbf}$$

$$X_c = \frac{L}{F}\int_0^{R_2-R_1}(P - P_{min})x\,dx,\ \text{in.}$$

$$x = x_1 + n\,\Delta x,\ \text{in.}$$

$$P(x) = \left\{\exp\left[-\frac{K_1}{2}\left(\frac{x + R_1}{R_2}\right)^2\right]\left[\frac{P_2^2\exp\left[\dfrac{K_1}{2}\right] - P_1^2\exp\left[\dfrac{K_1R_1^2}{2R_2^2}\right]}{\displaystyle\int_0^{R_2-R_1}\frac{\exp\left[\dfrac{K_1}{2}\left(\dfrac{x + R_1}{R_2}\right)^2\right]}{x + R_1}\,dx}\right.\right.$$
$$\left.\left.\times\int_0^x\frac{\exp\left[\dfrac{K_1}{2}\left(\dfrac{x + R_1}{R_2}\right)^2\right]}{x + R_1}\,dx + P_1^2\exp\left[\frac{K_1R_1^2}{2R_2^2}\right]\right]\right\}^{1/2}$$

$$\text{where } K_1 = -\frac{3R_2^2\Omega^2}{5\mathcal{R}T}$$

$$\bar{F} = \frac{F}{|P_1 - P_2|(R_2 - R_1)L}$$

$$\bar{X}_c = \frac{X_c}{R_2 - R_1}$$

$$\text{Torque} = \left[\frac{33\,000\ (\text{Power})}{N}\right]\left(\frac{12\ \text{in.}}{\text{ft}}\right),\ \text{ft-lbf}$$

Analytical Solution For Restricted Case

When

$$\left|\frac{K_1}{2}\right| = \left|-\frac{3R_2^2\Omega^2}{10\Re T}\right| < 1$$

the integral

$$\int \frac{\exp\left[\frac{K_1\tilde{X}^2}{2}\right]}{\tilde{X}}\, d\tilde{X}$$

can be evaluated by integrating an infinite series expansion. For most practical seal problems, the value of the parameter $-K_1/2$ is less than 1; thus, an analytical solution can be found for the pressure distribution and mass leakage. Now,

$$\int \frac{\exp\left[\frac{K_1\tilde{X}^2}{2}\right]}{\tilde{X}}\, d\tilde{X} = \int \left[\frac{1}{\tilde{X}} + \frac{K_1\tilde{X}}{2} + \frac{K_1^2\tilde{X}^3}{2^2 \cdot 2!} + \frac{K_1^3\tilde{X}^5}{2^3 \cdot 3!} + \cdots\right.$$

$$\left. + \left(\frac{K_1}{2}\right)^n \frac{\tilde{X}^{2n-1}}{n!} + \cdots\right] d\tilde{X} = l n\tilde{X} + L(\tilde{X}) + C$$

where

$$L(\tilde{X}) = \sum_{n=1}^{\infty} \frac{K_1^n(\tilde{X}^{2n} - 1)}{2^{n+1}\, n\, n!}$$

Thus, in dimensional form, the mass flow equation (24) becomes

$$\dot{M} = \frac{\pi h^3 \rho_2 \left\{ \exp\left[-\frac{K_1}{2}\left(\frac{R_1^2}{R_2^2} - 1\right)\right] P_2^2 - P_1^2 \right\} \exp\left(\frac{K_1}{2}\frac{R_1^2}{R_2^2}\right)}{12\,\mu P_2 \left[ln\left(\frac{R_1}{R_2}\right) + L\left(\frac{R_1}{R_2}\right)\right]} \tag{34}$$

The resulting squared pressure distribution equation becomes

$$P^2 = -\frac{ln\,\frac{r}{R_2} + L\left(\frac{r}{R_2}\right)}{ln\,\frac{R_1}{R_2} + L\left(\frac{R_1}{R_2}\right)} \exp\left[-\frac{K_1\left(\frac{r^2}{R_2^2} - \frac{R_1^2}{R_2}\right)}{2}\right] \left\{ \exp\left[-\frac{K_1}{2}\left(\frac{R_1^2}{R_2^2} - 1\right)\right] P_2^2 - P_1^2 \right\}$$

$$+ \exp\left[-\frac{K_1}{2}\left(\frac{r^2}{R_2^2} - 1\right)\right] P_2^2 \tag{35}$$

For no rotation, $K_1 = 0$; hence, $L(r/R^2) = 0$, and equation (34) becomes

$$\dot{M} = -\frac{\pi h^3 \rho_2 P_2 \left(1 - \frac{P_1^2}{P_2^2}\right)}{12\,\mu ln\,\frac{R_2}{R_1}} \tag{36}$$

And equation (34) becomes

$$\frac{P^2}{P_2^2} = 1 - \left(1 - \frac{P_1^2}{P_2^2}\right)\frac{R_2 - r}{R_2 - R_1} \tag{37}$$

Equation (36) and (37) are reducible to the form found in reference 3. Note that for the case when $\Delta R/R_1 << 1$, $ln(R_2/R_1) \cong -\Delta R/R_1$, and equation (36) becomes

$$\dot{M} = \frac{\pi h^3 R_1 \rho_2 P_2\left(1 - \frac{P_1^2}{P_2^2}\right)}{12\,\mu(R_2 - R_1)} \tag{38}$$

This is an identical form of the narrow slot leakage equation found in reference 5; thus, hydrostatic radial flow can be approximated with a narrow slot (plane flow) analysis with small error if $(R_2 - R_1) << R_1$.

Error in Estimating Mass Leakage by Hydrostatic Radial-Flow Formula

A formula will now be found to give the error that occurs in using the hydrostatic radial flow mass leakage equation when there is relative rotation of the sealing dam surfaces.

$$\frac{\Delta\dot{M}}{\dot{M}_{static}} = \frac{\dot{M}_{dynamic} - \dot{M}_{static}}{\dot{M}_{static}} = \frac{\text{Equation (24) - Equation (25)}}{\text{Equation (25)}}$$

$$= \frac{ln\,\frac{R_1}{R_2}\left\{\exp\left(\frac{K_1}{2}\right) - 1 + \frac{P_1^2}{P_2^2}\left[1 - \exp\left(\frac{K_1 R_1^2}{2R_2^2}\right)\right]\right\} + L\left(\frac{R_1}{R_2}\right)\left(\frac{P_1^2}{P_2^2} - 1\right)}{\left(1 - \frac{P_1^2}{P_2^2}\right)\left[ln\,\frac{R_1}{R_2} + L\left(\frac{R_1}{R_2}\right)\right]}$$

This formula can be further simplified by applying conditions present in most sealing dams:

(1) For $|K_1/2| < 1$,

$$\exp\left(\frac{K_1}{2}\right) \cong 1 + \frac{K_1}{2}$$

and

$$\exp\left(\frac{K_1R_1^2}{2R_2^2}\right) \cong 1 + \frac{K_1R_1^2}{2R_2^2}$$

(2) For $R_1 >> R_2 - R_1$,

$$ln\frac{R_2}{R_1} \cong -\frac{\Delta R}{R_1}$$

(3) $L(R_1/R_2) \cong K_1/4\left(R_1^2/R_2^2 - 1\right)$

These conditions yield the following formula:

$$\frac{\Delta\dot{M}}{\dot{M}_{static}} = \frac{\Delta R\left[1 - \frac{P_1^2}{P_2^2} - \frac{3R_2^2\Omega^2}{5\mathcal{R}T}\left(1 - \frac{P_1^2}{P_2^2}\frac{R_1^2}{R_2^2}\right)\right]}{R_1\left(1 - \frac{P_1^2}{P_2^2}\right)\left[\frac{\Delta R}{R_1} + \frac{3R_2^2\Omega^2}{5\mathcal{R}T}\left(\frac{R_1^2}{R_2^2} - 1\right)\right]} - 1 \tag{39}$$

DISCUSSION

The approximate formula given by equation (39) can be used to determine whether rotation is important in radial mass leakage calculations. If rotational effects are shown not to be important, the simpler hydrostatic flow formulas can be used. Figure 5 illustrates that there is excellent agreement between the approximate formula (eq. (39)), which includes only the first term of the convergent series, and the numerical solution using the computer program (ref. 8); radius ratios of 0.80 and 0.99 are shown for $-K_1/2 = 6\times10^{-6}$.

For laminar flow across an isothermal compressible flow sealing dam, the change in mass leakage due to relative surface rotation appears to be negligible for many cases where the radial pressure differential is large, the gap is small, and the rotational speed is moderate. This is illustrated by equation (39); two cases are evaluated numerically in appendix A. However, in a liquid medium (incompressible fluid), the centrifugal force in laminar radial flow can have a substantial effect because of the higher density. For

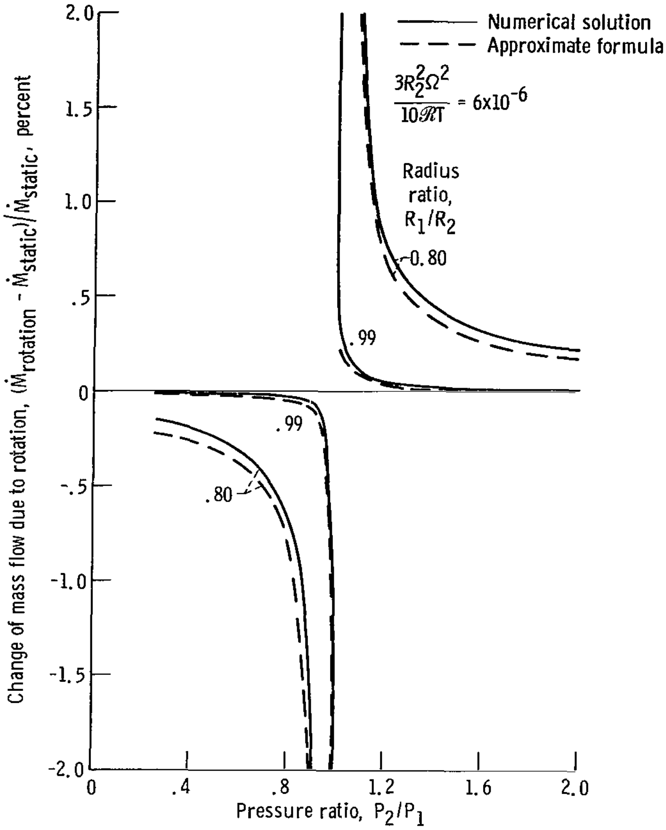


Figure 5. - Change of mass flow due to rotation as function of pressure ratio. Comparison of numerical solution with approximate formula.

example, water at room temperature is about three orders of magnitude more dense than air.

If a very small pressure differential exists across the sealing dam, the rotational effects on the radial flow are important even for moderate speeds. An example of such a case is given in appendix A, which also shows that the incompressible volume flow formula by Snapp (ref. 10) gives a fair approximation. The approximate formulas for estimating the effect of rotation on leakage for both compressible and incompressible fluids are as follows:

Incompressible flow analysis (from Snapp (ref. 10)):

$$\frac{\Delta Q}{Q_{static}} = \frac{3\rho\Omega^2\left(R_2^2 - R_1^2\right)}{20\left(P_1 - P_2\right)}$$

Compressible flow analysis:

$$\frac{\Delta \dot{M}}{\dot{M}_{static}} = \frac{\Delta R\left[1 - \frac{P_1^2}{P_2^2} + \frac{K_1}{2}\left(1 - \frac{P_1^2}{P_2^2}\frac{R_1^2}{R_2^2}\right)\right]}{R_1\left(1 - \frac{P_1^2}{P_2^2}\right)\left\{\frac{\Delta R}{R_1} - \frac{K_1}{4}\left[\left(\frac{R_1}{R_2}\right)^2 - 1\right]\right\}} - 1$$

where $K_1 = - 3R_2^2\Omega^2/5\mathcal{R}T$. Also, the rotational flow is always important for determining the power loss (due to viscous shearing) and for determining the transition to turbulent flow. The rotational flow component of the velocity may be responsible for the flow regime becoming turbulent in the circumferential direction (shear flow direction), which will mean the entire flow field (radial pressure flow) will be turbulent.

Figure 6 illustrates a simplified envelope of possible sealing dam flow regimes, showing the region in which this analysis is valid. For the hydrostatic case, the viscous flow analysis presented is valid for $Re_h(h/\Delta R) < 1$. The parameter governing the flow regime for radial flow was found to be the Mach number rather than the pressure flow Reynolds number. This is true for very small gaps since calculations indicate that the flow can become choked in the radial direction before the minimum transition pressure-flow Reynolds number is reached. Choking of the flow, of course, will occur when $M = 1$. The limit for this analysis to be valid is $M < 1/\sqrt{\gamma}$; beyond this, the isothermal

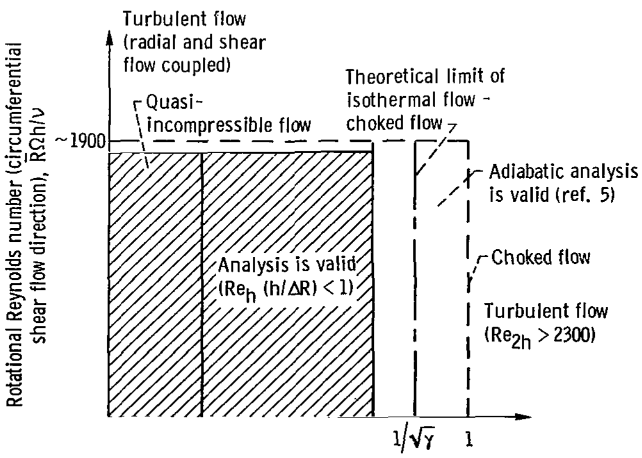


Figure 6. - Envelope of possible sealing dam flow regimes, illustrating region of analysis validity.

assumption no longer holds. (According to ref. 5 the adiabatic assumption is valid to $M = 1$.)

For radial pressure flow only, transition to turbulent flow will occur by Tollmien-Schlichting waves. The minimum transition Reynolds number Re_{2h} (based on an average velocity and hydraulic diameter) will be between 2300 and 6000; that is, between the minimum transition Reynolds number for fully developed pipe flow and what appears to be the theoretical minimum transition Reynolds number for plane Poiseuille flow (ref. 11) in infinitesimal disturbance theory. The radial flow situation of the sealing dam approximates the latter case because the aspect ratio used in the hydraulic diameter calculation is very large.

For the case of only circumferential shear flow with no imposed radial pressure gradient, the flow will remain laminar until a critical rotational Reynolds number is exceeded; then the flow will become turbulent. For this narrow gap sealing dam analysis ($\Delta R >> h$), the critical rotational Reynolds number for transition appears to be simple Couette flow transition Reynolds number. Thus, $Re_r = \bar{R}\Omega h/\nu \cong 1900$ for transition (ref. 12); and the smaller the gap, the longer transition speed is delayed. Thus, the wall has a stabilizing effect on the flow; and this should be considered in selecting the design gap (e.g., for a 1-ft-diam seal operating at room temperature, the transition speed would be 71 500 rpm for a 1-mil gap, but would be 179 000 rpm for a 0.4-mil gap).

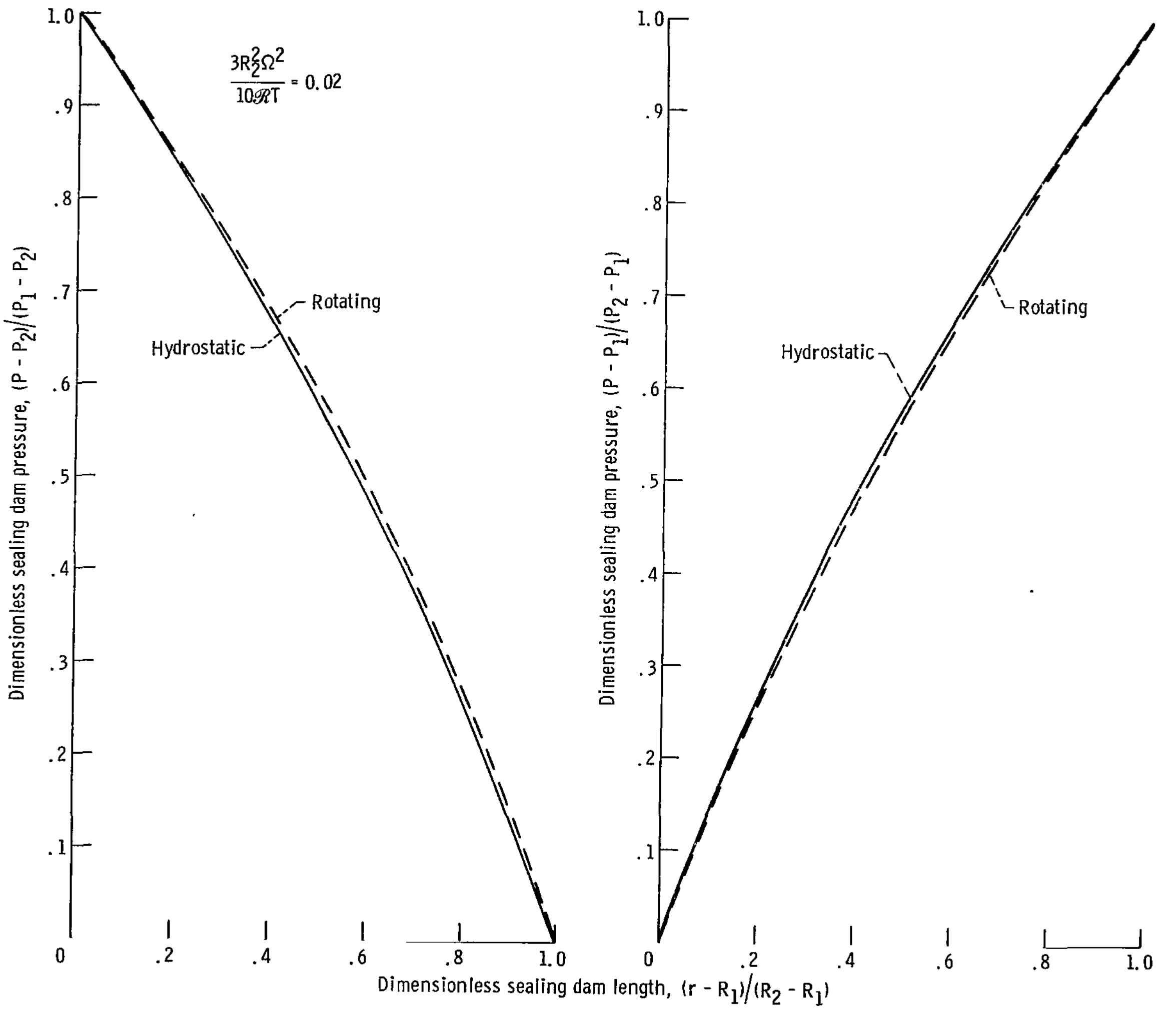
Most flow regime transitions have been experimentally observed for incompressible flow between a disk and a confined housing, but there is reason to believe that transition will also occur for a compressible fluid in this range of Reynolds number. Theodorsen and Regier (ref. 13) conducted an experimental investigation on a rotating disk in an infinite compressible fluid medium and found that transition and drag are independent of Mach number. The highest transition Reynolds number $\left(R_2^2\Omega^2/\nu\right)$ was 3×10^5 for the most highly polished disk. However, more experimental work is needed to find the transition Reynolds number for compressible flow between two disks in relative rotation.

In this analysis, which is valid for a modified Reynolds number $Re_h(h/\Delta R) << 1$, the circumferential velocity v can be found independently of the radial velocity u (i.e., v does not depend on u, but u depends on v (see eq. (13)). The radial velocity u is affected by the rotation through the centrifugal force component in the radial direction (see eq. (12)). In a higher-order analysis, when $Re_h(h/\Delta R)$ is no longer less than 1, the fluid velocity component in the circumferential direction will depend on the radial pressure flow, which will, in turn, depend on the rotating flow velocity component.

The region where the radial and the tangential flow are coupled involves the solution of two simultaneous nonlinear partial differential equations and is beyond the scope of this investigation. This more complex formulation would be necessary for a stability analysis and for a more detailed examination of the flow field. In turbulent flow, of course, both the circumferential and radial flow velocity components depend on each other.

Rotational Effects on Pressure and Velocity Profiles

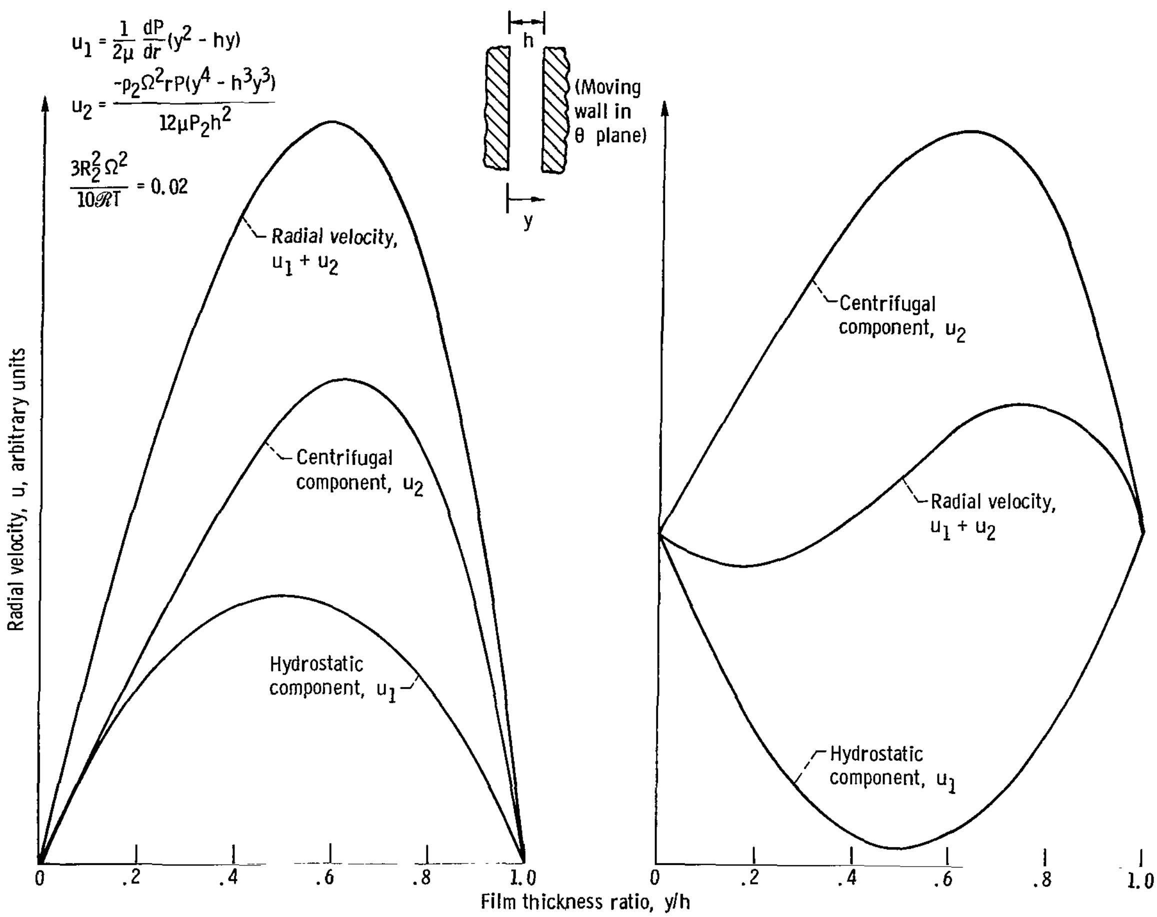
The pressure distribution in the radial direction was found from equation (25). The internally pressurized case is shown plotted in figure 7(a) for $-K_1/2 = 0.02$. Also shown in figure 7(a) is the hydrostatic radial pressure distribution, which was found from equation (25) with $\Omega = 0$. Note the sealing dam force is greater for rotation. The radial pressure distribution for the externally pressurized case is shown in figure 7(b). Here the sealing dam force is less than for the hydrostatic case. The radial pressure distribution for the case of equal inner and outer cavity pressures (similar to some bearing boundary conditions) and $-K_1/2 \leq 1$ was found to be a constant.



(a) Internally pressurized case.

(b) Externally pressurized case.

Figure 7. - Effect of rotation on radial pressure distribution.



(a) Internally pressurized case. Pressure ratio, $P_1/P_2 = 1.2$.

(b) Externally pressurized case. Pressure ratio, $P_1/P_2 = 0.833$.

Figure 8. - Radial velocity profile, moving wall in θ-plane. Radius ratio, $R_1/R_2 = 0.99$.

The radial velocity profiles for both internally and externally pressurized cases were found for $R_1/R_2 = 0.99$, $P_1/P_2 = 1.2$, and $-K_1/2 = 0.02$ and are shown in figures 8(a) and (b). These radial velocity profiles were obtained from equation (30). Also shown are the hydrostatic and centrifugal force velocity components. As expected, the centrifugal force component is effective in reducing the leakage in the externally pressurized case. The maximum velocity does not occur at the midgap position for the centrifugal force velocity component as occurs for the hydrostatic radial velocity component. The shear flow (circumferential direction) velocity profile is the classical simple Couette flow profile shown in figure 9.

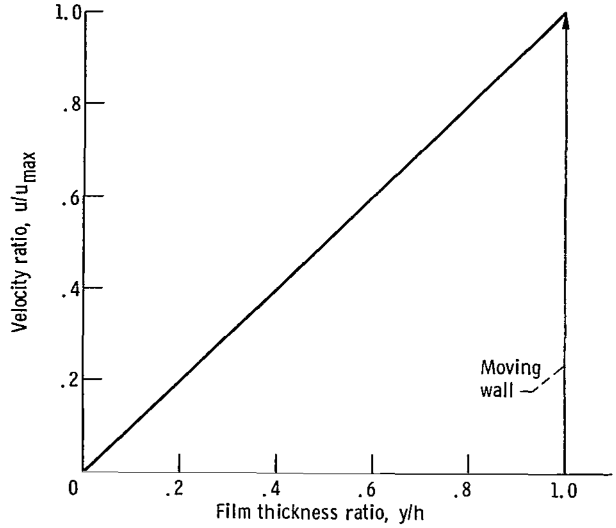


Figure 9. - Circumferential direction velocity profile (simple Couette flow).

SUMMARY OF RESULTS

The flow across a parallel sealing dam is analyzed for steady, laminar, subsonic, isothermal compressible flow with rotation. The analysis considered the effect of sealing dam rotation on mass leakage, pressure distribution, velocity distribution, and net pressure force; the hydrostatic case was used for comparison. The following pertinent results were obtained:

1. Rotation has significant effects on radial velocity, pressure distribution, center of pressure and mass flow at high speeds. When the radial pressure differential is very small and the speed is moderate, rotational effects are also important. (The seal example analyzed has a 17-percent increase in calculated leakage for a 0.2-psi pressure differential at a moderate speed.)

2. A hydrostatic analysis of the radial pressure flow is a good approximation of the rotating sealing dam case under the conditions that the ratio of the reference rotational velocity to reference radial pressure flow velocity must be less than the inverse square root of modified Reynolds number. (The seal example analyzed had a 0.1-percent deviation from the hydrostatic case.)

3. The error in estimating the mass leakage by using the hydrostatic radial flow formula was analyzed, and pertinent error analysis equations are given.

4. The region of validity of the analysis was defined as follows:

a. The analysis is valid until transition of the circumferential shear flow to turbulence.

b. The analysis is valid until the radial pressure flow approaches a Mach number of $1/\sqrt{\gamma}$, where γ is the ratio of specific heats, or until the radial pressure flow becomes turbulent.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 19, 1969,
120-27-04-90-22.

APPENDIX A

EXAMINATION OF CENTRIFUGAL FORCE EFFECT ON RADIAL AIRFLOW

Examine the rotational Reynolds number

$$Re_r = \frac{\overline{R}\Omega h}{\nu}$$

Let $\nu = 2\times10^{-4}$ square feet per second, $T = 500^{\circ}$ R, $\overline{R} = 1/2$ foot, $\mathcal{R} = 2\times10^{3}$ square feet per second squared per $^{\circ}$R, N = 2300 rpm (this can be considered a moderate speed), and h = 0.001 inch. Hence, $Re_r \cong 25$, much less than the critical transition Reynolds number for simple Couette flow, which is about 1900.

Now,

$$\left|\frac{K_1}{2}\right| = \left| - \frac{3R_2^2\Omega^2}{10\mathcal{R}T}\right| \cong 0.004$$

Since $|K_1/2| < 1$, equation (39) can be used to find the change in mass leakage caused by rotation

$$\frac{\Delta\dot{M}}{\dot{M}_{static}} \cong \left\{ \frac{\Delta R\left[1 - \dfrac{P_1^2}{P_2^2} - \dfrac{3R_2^2\Omega^2}{5\mathcal{R}T}\left(1 - \dfrac{P_1^2}{P_2^2}\dfrac{R_1^2}{R_2^2}\right)\right]}{R_1\left(1 - \dfrac{P_1^2}{P_2^2}\right)\left[\dfrac{\Delta R}{R_1} + \dfrac{3R_2^2\Omega^2}{5\mathcal{R}T}\left(\dfrac{R_1^2}{R_2^2} - 1\right)\right]} - 1 \right\}$$

Case I - Very Small Pressure Differential

When

$$\left.\begin{aligned} P_1 &= 100.2 \text{ psia}, \; R_1 = 5.70 \text{ in.} \\ P_2 &= 100 \text{ psia}, \; R_2 = 6.30 \end{aligned}\right\} \text{ at } \overline{R} = 1/2 \text{ ft}$$

equation (39) yields $\Delta \dot{M}/\dot{M}_{static} \cong 17$ percent.

Hence, rotation has a substantial effect on mass leakage when a very small pressure differential exists; however, in this example, the radial flow Mach number is very small, and the incompressible $\Delta Q/Q$ formula (pp. 22 and 23) predicts about a 25-percent change.

Case II - Moderate Pressure Differential

When

$$P_1 = 100 \text{ psia}, \; R_1 = 5.70 \text{ in.}$$

$$P_2 = 150 \text{ psia}, \; R_2 = 6.30 \text{ in.}$$

equation (35) yields

$$\frac{\Delta \dot{M}}{\dot{M}_{static}} \cong -0.1 \text{ percent}$$

Thus, the above example illustrates that centrifugal force does not have an appreciable effect on air mass leakage for moderate and large pressure ratios and moderate rotational speeds.

APPENDIX B

SYMBOLS

A	cross-sectional area, in.^{2}; m^2
a	speed of sound, ft/sec; m/sec
C	constant of integration
C_p	specific heat at constant pressure, Btu/(lbm)($^{\circ}$R); J/(kg)(K)
C_v	specific heat at constant volume, Btu/(lbm)($^{\circ}$R); J/(kg)(K)
D/Dt	material derivative, $\partial/\partial t + u(\partial/\partial r) + (v/r)(\partial/\partial\theta) + w(\partial/\partial z)$
F	sealing dam force, lbf; N
$\vec{F}$	body force vector
$\overline{F}$	dimensionless force, $F/(P_2 - P_1)(R_2 - R_1)L$
h	film thickness, nominal, in.; m
K_1	$-3R_2^2\Omega^2/5\mathcal{R}T$
L	sealing dam width, in.; m
$L(X)$	$\sum_{n=1}^{\infty} \frac{K_1^n(X^{2n} - 1)}{2^{n+1}\, n\, n!}$
M	Mach number
$\dot{M}$	mass flow, lbm/min; kg/sec
$\Delta\dot{M}$	change in mass flow
$\dot{M}^*$	dimensionless mass flow, $12\mu\dot{M}\mathcal{R}T/\pi h^3 P_2$
m	molecular weight of gas, lbm/lb-mole; kg/kg-mole
n	an integer
P	static pressure, psi; N/m^2
ΔP	pressure differential, psi; N/m^2
P_{min}	smaller pressure of two pressure boundary conditions, psi; N/m^2
P^*	dimensionless pressure, $P/\rho_0 U^2$

$\tilde{P}^2$ dimensionless P^2, P^2/P_2^2

Q net volume flow rate, st. cu ft/min; st. cu m/sec

$\underline{R}$ universal gas constant, 1545.4 ft-lbf/(lb-mole) (^{O}R); 8.3143 J/(kg)(mole)(K)

$\bar{R}$ mean radius, $(R_1 + R_2)/2$, in.; m

ΔR sealing dam length, $R_2 - R_1$, in.; m

$\mathcal{R}$ gas constant, $\underline{R}/m$, ft-lbf/(lbm)(^{O}R); J/(kg)(K)

Re_h Reynolds number in radial direction, $\rho Uh/\mu$

Re_r Reynolds number in circumferential direction, $\rho\bar{R}\Omega h/\mu$

r radial direction coordinate

r^* dimensionless radial coordinate, $r/\Delta R$

$\tilde{r}$ normalized radial coordinate, $(r - R_2)/R_2$

T temperature, ^{O}F; K

$\bar{T}$ average temperature, ^{O}F; K

U pressure flow reference velocity, ft/sec; m/sec

u velocity in r-direction or x-direction, ft/sec; m/sec

u^* dimensionless velocity, u/U

V reference shear flow velocity, ft/sec; m/sec

v velocity in θ-direction, ft/sec; m/sec

v^* dimensionless velocity, v/V

w velocity in y-direction, ft/sec; m/sec

w^* dimensionless velocity, w/W_{ref}

W reference velocity across film thickness, $U(h/\Delta R)$, ft/sec; m/sec

$\tilde{X}$ transformed coordinate, $1 + \tilde{r}$

X_c center of pressure in radial or X-direction, in.; m

$\bar{X}_c$ dimensionless center of pressure, $X_c/(R_2 - R_1)$

x coordinate in pressure gradient direction

y coordinate across film thickness

y^* dimensionless coordinate, y/h

z shear flow coordinate in Cartesian system

γ specific-heat ratio, C_p/C_v

θ circumferential coordinate

λ second viscosity coefficient or coefficient of bulk viscosity

μ absolute or dynamic viscosity, (lbf)(sec)/in.^{2}; (N)(sec)/m^2

ν kinematic viscosity, ft^2/sec; m^2/sec

ρ density, (lbf)(sec^2)/in.^{4}; kg/m^3

ρ^* dimensionless density, ρ/ρ_0

Ω angular rotational velocity, rad/sec

$\vec{\nabla}$ Del operator, $(\partial/\partial r)\hat{i} + (1/r)(\partial/\partial\theta)\hat{j} + (\partial/\partial z)\hat{k}$

Subscripts:

av average

h based on film thickness

r based on radius

0 reference

1 inner radius or inlet

2 outer radius or exit

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